Meson-baryon couplings and the F/D ratio from QCD sum rules

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Abstract

Motivated by the recent work on the calculation of the πNN coupling constant using QCD sum rule beyond the chiral limit, we construct the corresponding sum rules for the couplings, ηNN, $\pi \Xi \Xi$, $\eta \Xi \Xi$, $\pi \Sigma \Sigma$ and $\eta \Sigma \Sigma$. In constructing the η-baryon sum rules, we use the second moment of the η wave function, which we obtain from the pion wave function after SU(3) rotation. In the SU(3) symmetric limit, we can identify the term responsible for the F/D ratio in the OPE, which after the sum rule analysis gives $F/D \sim 0.2$, a factor of 3 smaller than from other studies. We also present a qualitative analysis including the SU(3) breaking terms.

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I. INTRODUCTION

Meson-baryon couplings in the SU(3) sector are important quantities to be determined in modern nuclear physics. Each coupling, representing the strength of baryon-baryon interaction to each mesonic channel, is an important ingredient for a systematic analysis of baryon-baryon scatterings and meson productions off a baryon. Experimentally, the couplings are determined by fitting the experimental data of meson-baryon scatterings, baryon-baryon phase shift analysis [1,2]. However, in general, it is difficult to pin down a specific channel from a certain reaction and determine unambiguously the coupling of concern. In addition, hadronic models used in fitting processes are not often unique, which provides an additional limitation of the conventional methods.

SU(3) symmetry, as it provides a systematic classification of mesons and baryons, is expected to govern the meson-baryon couplings. Indeed, according to Ref. [3], all the meson-baryon couplings in the SU(3) limit satisfy simple relations represented by the πNN coupling, which is well-known experimentally, and the F/D ratio. This systematic classification of the couplings is a basis for making realistic potential models for hyperon-baryon interactions [2]. In this approach, the F/D ratio is an input for the analysis, usually determined from other sources for example the SU(6) consideration. However, a more realistic and self-contained method is to determine the ratio within the SU(3) classification. One such method is to invoke QCD sum rules [4] and calculate the meson-baryon couplings in the SU(3) limit. This not only provides a QCD prediction for the F/D ratio but also gives insights as to how the couplings should be constrained even in the SU(3) breaking case.

QCD sum rules [4] utilize the two aspects of QCD, perturbative and nonperturbative effects, in representing hadronic spectral properties in terms of QCD parameters. Within this framework, a correlation function of QCD operators is evaluated via the operator product expansion (OPE). This expansion includes the perturbative part as well as the nonperturbative effects, which are systematically included by the power corrections. In the hadronic side, an ansatz for the correlator is introduced in terms of hadronic degrees of freedom and it is matched with the OPE. The matching provides the hadronic parameter of concern in terms of QCD parameters. QCD sum rules have been widely used [5] to predict baryonic masses, hadronic coupling constants and so on. In some cases, they are known to be successful even though some cares must be taken in constructing a sum rule [6,7].

When QCD sum rules are applied to meson-baryon couplings, a good place for testing QCD sum rules is to calculate the πNN coupling $g_{\pi N}$ [5,6,8–10] as its value is relatively well-known experimentally. A successful reproduction of this coupling within QCD sum rules may provide a solid framework to extend its method to other (not well-known) meson-baryon couplings. Indeed, in a series of papers [9,6,10], the πNN coupling has been calculated using the correlation function of the nucleon interpolation field J_N,

$$\Pi(q,p) = i \int d^4x e^{iq \cdot x} \langle 0|T[J_N(x)\bar{J}_N(0)]|\pi(p)\rangle \ . \tag{1}$$

This two-point correlation function with a pion seems to be more suitable than the three-point function approaches [11] because we do not need to worry about the contribution from the higher resonances $\pi(1300)$ and $\pi(1800)$ which could introduce substantial errors in determining the pion-nucleon coupling [12]. Moreover, using the two-point correlation

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function, the sum rule can be straightforwardly extended to other meson-baryon couplings and the SU(3) symmetric limit can be easily recovered.

One of us [10] has recently proposed to construct a QCD sum rule starting from Eq. (1) and going beyond the chiral limit. There [10], a QCD sum rule for the $i\gamma_5$ structure at the $p^2 = m_\pi^2$ order is constructed. One interesting observation made in Ref. [10] is that the quark mass terms included as a consistent chiral counting are found to be important in stabilizing the sum rule and crucial in producing πNN coupling close to its empirical one. The uncertainties from QCD parameters in the extracted $g_{\pi N}$ are estimated to be around ± 1, sensitivity to the continuum threshold is found to be small, and the unknown single-pole term which appears due to the transition of $N \to N^*$ is also estimated to be small. Hence, this sum rule beyond the chiral limit seems to have nice features and may provide a reliable framework for extending to other meson-baryon couplings.

In this work, we apply the QCD sum rules beyond the chiral limit to other meson-baryon couplings in the SU(3) sector. By keeping the SU(3) symmetry in constructing the sum rules, we can compare the OPE of each sum rule to the SU(3) relation of the coupling, which allows us to identify the OPE terms generating the F/D ratio. Then a subsequent Borel analysis of the sum rule determines the F/D ratio. Reliability of this value can be checked by further analyzing other meson-baryon sum rules. Furthermore, the sum rules at the SU(3) limit will give us a hint how the SU(3) symmetry breaking is reflected in the couplings. By identifying differences in the OPE from each sum rule, we might be able to predict how each OPE term should be modified as we switch on the SU(3) breaking effect. Even though we do not precisely know the OPE in the SU(3) breaking limit, we may still see a general trend on how the physical couplings should be, which can act as constraints to be satisfied in experimental analysis.

For this purpose, we reconstruct QCD sum rules for the πNN coupling and extend the framework to ηNN, $\pi \Xi\Xi$, $\eta \Xi\Xi$, $\pi \Sigma\Sigma$, $\eta \Sigma\Sigma$. One of these extended sum rules will be compared to the πNN sum rule and used to estimate the F/D ratio. The other sum rules provide consistency checks for this value within the same framework. In the case of the η-baryon sum rules, we assume the second moment of the twist-3 η wave function to be the same as the pion case. This assumption is supported by the OPE satisfying the classification suggested from the SU(3) symmetry. We then speculate qualitatively how the SU(3) breaking is reflected in the couplings.

The paper is organized as follows. In Section II, we reconstruct the QCD sum rule for the πNN coupling beyond the chiral limit. We apply the similar sum rule to the ηNN coupling in Section III. We construct the sum rules for $\pi \Xi\Xi$ and $\eta \Xi\Xi$ in Section IV, $\pi \Sigma\Sigma$ and $\eta \Sigma\Sigma$ in Section V. In Section VI, we present our numerical analysis for the couplings in the SU(3) limit and provide constraints for the F/D ratio. In Section VII, we qualitatively study how the SU(3) breaking appears in the couplings. We summarize in Section VIII.

II. QCD SUM RULES FOR PION-NUCLEON COUPLING

In this section, we construct a QCD sum rule for πNN coupling beyond the chiral limit. The content of this part can be found in Ref. [10] but we present this sum rule again with some technical details that can be straightforwardly extended for other meson-baryon couplings in later sections.

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We consider the two-point correlation function with a pion,

$$\Pi(q,p) = i \int d^4 x e^{iq \cdot x} \langle 0|T[J_p(x)\bar{J}_p(0)]|\pi^0(p)\rangle \equiv \int d^4 x e^{iq \cdot x} \Pi(x,p) \ . \tag{2}$$

J_p is the proton interpolating field suggested by Ioffe [13],

$$J_p = \epsilon_{abc}[u_a^T C \gamma_\mu u_b]\gamma_5 \gamma^\mu d_c \ , \tag{3}$$

where a, b, c are color indices, T denotes the transpose with respect to the Dirac indices, C the charge conjugation. From this correlator, we collect the terms contributing to the $i\gamma_5$ Dirac structure and expand them in terms of the pion momentum p^μ. The correlator in this expansion takes the form,

$$\Pi_0(q^2) + p \cdot q \Pi_1(q^2) + p^2 \Pi_2(q^2) + \cdots \ . \tag{4}$$

As the nucleon momentum q_μ is independent from the pion momentum p_μ, each scalar function can be used to construct a QCD sum rule. The correlator at the soft-pion limit Π_0 is equivalent to the nucleon chiral-odd sum rule [8]: it does not provide an independent determination of the coupling. The OPE contributing to Π_1 is basically the same as the Π_0 sum rule, again not useful for calculating the coupling. We therefore consider Π_2 in constructing a sum rule. By putting the pion on its mass-shell $p^2 = m_\pi^2$, we construct a sum rule beyond the chiral limit, which will provide a prediction for the pion-nucleon coupling $g_{\pi N}$ beyond the chiral limit. As we will see, it is at this order that each meson-baryon sum rule is distinct from the other sum rules.

In constructing the phenomenological side, we replace the nucleon interpolating field with the physical nucleon field, $J_p \rightarrow \lambda_N \psi_p$, and, using the pseudoscalar Lagrangian, $g_{\pi N}\bar{\psi}_p i\gamma_5 \psi_p \pi^0$, we evaluate the correlator in terms of hadronic degrees of freedom. At the chiral order $p^2 = m_\pi^2$ from the correlator containing the $i\gamma_5$ Dirac structure, the phenomenological correlator takes the form

$$m_\pi^2 \Pi_2^{phen}(q^2) = m_\pi^2 \frac{g_{\pi N}\lambda_N^2}{(q^2 - m_N^2)^2} + \cdots \ . \tag{5}$$

The ellipses denote the contributions when J_p couples to higher resonances. This includes the continuum contribution whose spectral density is parametrized by a step function with a certain threshold S_0 and the single-pole terms associated with the transitions $N \rightarrow N^*$ [14]. Here m_N denotes the nucleon mass.

In the OPE, we only keep the quark-antiquark component of the pion wave function and use the vacuum saturation hypothesis to factor out higher dimensional operators in terms of the pion wave function and the vacuum expectation values. Accordingly, it is straightforward to write the correlator in the coordinate space,

$$\Pi(x,p) = -i\epsilon_{abc}\epsilon_{a'b'c'}\Big\{ \quad \gamma_5 \gamma^\mu D_{cc'}^d \gamma^\nu \gamma_5 \text{Tr}\left[iS_{aa'}(x)(\gamma_\nu C)^T iS_{bb'}^T(x)(C\gamma_\mu)^T\right]$$

$$- \gamma_5 \gamma^\mu D_{cc'}^d \gamma^\nu \gamma_5 \text{Tr}\left[iS_{ab'}(x)\gamma_\nu C iS_{ba'}^T(x)(C\gamma_\mu)^T\right]$$

$$- \gamma_5 \gamma^\mu iS_{cc'}(x)\gamma^\nu \gamma_5 \text{Tr}\left[iS_{ab'}(x)\gamma_\nu C (D_{ba'}^u)^T(C\gamma_\mu)^T\right]$$

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$$+ \gamma_5\gamma^\mu iS_{cc'}(x)\gamma^\nu\gamma_5\text{Tr}\left[iS_{aa'}(x)(\gamma_\nu C)^T(D^u_{bb'})^T(C\gamma_\mu)^T\right]$$

$$+ \gamma_5\gamma^\mu iS_{cc'}(x)\gamma^\nu\gamma_5\text{Tr}\left[D^u_{aa'}(\gamma_\nu C)^T iS^T_{bb'}(x)(C\gamma_\mu)^T\right]$$

$$\left. - \gamma_5\gamma^\mu iS_{cc'}(x)\gamma^\nu\gamma_5\text{Tr}\left[D^u_{ab'}\gamma_\nu CiS^T_{ba'}(x)(C\gamma_\mu)^T\right]\right\} . \tag{6}$$

The quark propagators $iS(x)$ inside the traces are the u-quark propagators and the ones outside of the traces are the d-quark propagators. Since we are interested in the $i\gamma_5$ Dirac structure, in most OPE we can replace the quark-antiquark component with a pion as follows,

$$(D^u_{aa'})^{\alpha\beta} \equiv \langle 0|u^\alpha_a(x)\bar{u}^\beta_{a'}(0)|\pi^0(p)\rangle \ \rightarrow \ \frac{\delta_{aa'}}{12}(i\gamma_5)^{\alpha\beta}\langle 0|\bar{u}(0)i\gamma_5 u(x)|\pi^0(p)\rangle . \tag{7}$$

A similar relation holds for the d-quark component. Contributions from the other Dirac structures, $\gamma_5\gamma_\mu$ and $\gamma_5\sigma_{\mu\nu}$, to this quark-antiquark component do not participate to the $i\gamma_5$ sum rule at the chiral order that we are considering.

We are concerned with the sum rule for the $i\gamma_5$ structure at the order $p^2 = m^2_\pi$. The p_μ dependence appears only in the quark-antiquark component. Obviously, the second order terms in the expansion of the pion matrix element in p_μ should contribute to the sum rule. In a consistent approach at this order, terms linear in quark mass (m_q) should also be included as m_q is the same chiral order with m^2_π via the Gell-Mann−Oakes−Renner relation,

$$- 2m_q\langle\bar{q}q\rangle = m^2_\pi f^2_\pi . \tag{8}$$

These terms are obtained by taking m_q terms from a quark propagator and at the same time taking the zeroth moment of the pion matrix element. Additional contributions at this order are from the three-particle wave function, obtained by taking a gluon tensor from a quark propagator and moving into the quark-antiquark component,

$$\langle 0|g_s G^A_{\mu\nu}(0)d^\alpha_a(x)\bar{d}^\beta_b(0)|\pi^0(p)\rangle \rightarrow \frac{if_{3\pi}}{32\sqrt{2}}m^2_\pi t^A_{ab}(\gamma_5\sigma_{\mu\nu})^{\alpha\beta} , \tag{9}$$

where $f_{3\pi} = 0.003$ GeV^2 [15] and the color matrices t^A are normalized $tr(t^A t^B) = \delta^{AB}/2$.

With these in mind, we calculate the correlator Eq. (6) using the quark propagator given in Ref. [16][1]. The OPE up to dimension 8 in the coordinate space is

$$+ \frac{2i}{\pi^4}\langle 0|\bar{d}(0)i\gamma_5 d(x)|\pi^0\rangle\frac{1}{x^6} - \frac{3if_{3\pi}m^2_\pi}{4\sqrt{2}\pi^4}\frac{1}{x^4}$$

$$- \frac{i}{96\pi^2}\left\langle\frac{\alpha_s}{\pi}\mathcal{G}^2\right\rangle\langle 0|\bar{d}(0)i\gamma_5 d(x)|\pi^0\rangle\frac{1}{x^2} - \frac{i}{2\pi^2}m_u\langle\bar{u}u\rangle\langle 0|\bar{d}(0)i\gamma_5 d(x)|\pi^0\rangle\frac{1}{x^2}$$

$$+ \frac{im_u\langle\bar{d}g_s\sigma\cdot\mathcal{G}d\rangle + im_d\langle\bar{u}g_s\sigma\cdot\mathcal{G}u\rangle}{48\pi^2}\langle 0|\bar{u}(0)i\gamma_5 u(x)|\pi^0\rangle\left[ln(\frac{-x^2\Lambda^2_c}{4}) + 2\gamma_{EM}\right] . \tag{10}$$

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[1]Note, the gluon strength tensor used in Ref. [16] has opposite sign from the one in Ref. [5]. This difference is simply due to how the covariant derivative is defined.

Except for the last term containing the quark-gluon mixed condensate, all others come from the first two terms in Eq. (6). The other four terms in Eq. (6), as they have the quark-antiquark component inside of the traces, contribute mostly zero to the OPE except producing the last term in Eq. (10). Note, only the last term contains the u-quark component with a pion while the others contain the d-quark component. This identification will be useful for later developments. Furthermore, as the last term contains the quark mass m_q which is the chiral order that we are considering, we need to take only the local contribution to the u-quark component with a pion (the part that survives in the soft-pion limit.). Under the Fourier transformation to the momentum space, the cutoff mass Λ_c and Euler-Mascheroni constant γ_{EM} will disappear.

The pion matrix element appearing in the OPE can be written in terms of the twist-3 pion wave function,

$$\langle 0|\bar{u}(0)i\gamma_5 u(x)|\pi^0(p)\rangle = -\frac{\langle\bar{u}u\rangle}{f_\pi}\int_0^1 dt e^{-itp\cdot x}\varphi_p(t) ,$$

$$\langle 0|\bar{d}(0)i\gamma_5 d(x)|\pi^0(p)\rangle = +\frac{\langle\bar{d}d\rangle}{f_\pi}\int_0^1 dt e^{-itp\cdot x}\varphi_p(t) . \tag{11}$$

The zeroth and the second moment of this twist-3 pion wave function are needed in constructing our sum rule. Note that the overall normalization of the matrix element is fixed by the soft-pion theorem, which gives opposite signs between the d-quark and the u-quark component. Also, the soft-pion theorem fixes the zeroth moment of the pion wave function to $\int_0^1 dt\varphi_p(t) = 1$. The twist-3 wave function is determined uniquely if the three-particle wave function is known [17]. However, the three-particle wave function gives only small corrections to the asymptotic form of the twist-3 wave function $[\varphi_p(t) = 1]$. Therefore, the second moment obtained from the asymptotic wave function, which is fixed to $\int_0^1 dt\, t^2\varphi_p(t) = 1/3$, is not so different from the moment using more realistic wave function [15,18].

Using the zeroth moment of the wave function for the OPE containing m_q and the second moment for the rest of the OPE (except for the term coming from the three-particle wave function), we obtain after Fourier transformations,

$$m_\pi^2\Pi_2^{ope}(q^2) = m_\pi^2 ln(-q^2)\left[\frac{\langle\bar{q}q\rangle}{12\pi^2 f_\pi} + \frac{3f_{3\pi}}{4\sqrt{2}\pi^2}\right] - 2m_q\langle\bar{q}q\rangle^2\frac{1}{f_\pi q^2}$$

$$+ m_\pi^2\frac{1}{72f_\pi}\langle\bar{q}q\rangle\left\langle\frac{\alpha_s}{\pi}\mathcal{G}^2\right\rangle\frac{1}{q^4} + \frac{2}{3}m_q m_0^2\langle\bar{q}q\rangle^2\frac{1}{f_\pi q^4} . \tag{12}$$

Here the quark-gluon mixed condensate is parametrized as $\langle\bar{d}g_s\sigma\cdot\mathcal{G}d\rangle \equiv m_0^2\langle\bar{d}d\rangle$ and similarly for the u-quark with $m_0^2 = 0.8$ GeV^2. In obtaining Eq. (10), we have taken the isospin symmetric limit, $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle \equiv \langle\bar{q}q\rangle$ and $m_u = m_d \equiv m_q$.

We now match the OPE with its phenomenological counterpart [Eq. (5)] using the single-variable dispersion relation [7] [2]. Under the Borel transformation with respect to $-q^2$, we obtain the sum rule,

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[2] In QCD sum rules with external fields, the double dispersion relation has been proposed as a proper representation of the correlator [19]. As the correlator in the tree level contains two baryonic

$$g_{\pi N} m_\pi^2 \lambda_N^2 e^{-m_N^2/M^2}[1 + AM^2] =$$
$$-m_\pi^2 M^4 E_0(x) \left[\frac{\langle \bar{q}q \rangle}{12\pi^2 f_\pi} + \frac{3f_{3\pi}}{4\sqrt{2}\pi^2} \right] + \frac{2m_q}{f_\pi} \langle \bar{q}q \rangle^2 M^2 + \frac{m_\pi^2}{72 f_\pi} \langle \bar{q}q \rangle \left\langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \right\rangle + \frac{2m_q}{3f_\pi} m_0^2 \langle \bar{q}q \rangle^2$$
$$\equiv \mathcal{O}_1 + \mathcal{O}_2 + \mathcal{O}_3 + \mathcal{O}_4 . \tag{13}$$

In the RHS, only the last term comes from the u-quark component with a pion, all others come from the d-quark component. Note that $\mathcal{O}_2$ and $\mathcal{O}_4$ contain the quark-mass. They are included as a consistent chiral counting. The contribution from $N \to N^*$ [14] is denoted by the unknown constant A. The continuum contribution is included by the factor, $E_n(x \equiv S_0/M^2) = 1 - (1 + x + \cdots + x^n/n!)e^{-x}$ where S_0 is the continuum threshold. The quark-mass dependence can be converted to the m_π^2 dependence via the Gell-Mann−Oakes−Renner relation, which then can be taken out from both sides as an overall factor. This sum rule, when combined with the nucleon chiral odd sum rule, gives a reliable value for the the pion-nucleon coupling $g_{\pi N}$ with small uncertainty and very closed to its empirical value [10].

Before closing this section, we comment about the sign of each contribution in the OPE Eq. (13). That is, $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_4$ contribute with the same sign, while $\mathcal{O}_3$ contributes with the opposite sign. However, the magnitude of $\mathcal{O}_3$ is only 7% of $\mathcal{O}_4$. Therefore, most OPE terms add up in producing the πNN coupling.

III. QCD SUM RULES FOR ηNN COUPLING

Motivated by the πNN sum rule beyond the chiral limit, we construct in this section a sum rule for the ηNN coupling. As in the πNN sum rule calculation, we consider the $i\gamma_5$ structure from the correlator,

$$i \int d^4 x e^{iq \cdot x} \langle 0|T[J_p(x)\bar{J}_p(0)]|\eta(p)\rangle , \tag{14}$$

at the order $p^2 = m_\eta^2$. In the followings, we neglect the mixing between η-η' and therefore η is assumed to be η_8. This is exact in the SU(3) limit and it does not affect our determination of the F/D ratio below.

The phenomenological side can be constructed as before by expanding the correlator in p_μ and collecting terms proportional to $p^2 = m_\eta^2$. The phenomenological side takes the form

$$m_\eta^2 \frac{g_{\eta N} \lambda_N^2}{(q^2 - m_N^2)^2} + \cdots . \tag{15}$$

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propagators, it seems reasonable to use the double dispersion relation. This is however misleading because the double dispersion relation produces spurious terms coming from subtraction terms which should not contribute to QCD sum rules. This is because the spectral density obtained from the double dispersion relation is not compatible with the duality assumption used for modeling the continuum contribution [7]. When the spurious terms are subtracted out from the sum rule using the double dispersion relation, the resulting sum rule is equivalent to the one using the single dispersion relation.

Note that the coupling $g_{\eta N}$ in this expansion is defined at the kinematical point $p^2 = 0$, that is, $g_{\eta N}(p^2 = 0)$. But the physical coupling is defined at $p^2 = m_\eta^2$. The correction of this kind is negligible in the πNN case. However, in the present case where m_η^2 is much heavier than m_π^2, one might expect some corrections from the form factor. The monopole type form factor is often used for meson-baryon couplings [1]. In the present case, the coupling should be written

$$g_{\eta N}(p^2) = g_{\eta N} \frac{\Lambda^2 - m_\eta^2}{\Lambda^2 - p^2} , \tag{16}$$

with $\Lambda \sim 1.5$ GeV according to Bonn potential [1]. By expanding the form factor in p^2, we have

$$\frac{\Lambda^2 - m_\eta^2}{\Lambda^2 - p^2} = \left(1 - \frac{m_\eta^2}{\Lambda^2}\right) + p^2 \frac{\Lambda^2 - m_\eta^2}{\Lambda^4} + \cdots . \tag{17}$$

The coupling $g_{\eta N}$ appeared in Eq. (15) is the physical coupling multiplied by the first term in this expansion, which will be determined in this method. The correction to $g_{\eta N}$ as we move from $p^2 = 0$ to $p^2 = m_\eta^2$ is $m_\eta^2/\Lambda^2 \sim 0.13$. Therefore, from this form factor effect, the physical coupling should be larger by 13 % than what we will determine in this work. Of course, more softer cut-off like $\Lambda \sim 1$ GeV will increase the physical coupling slightly more and we will discuss the uncertainty coming from Λ further in Section VII. The second term in Eq. (17) is combined with the correlator at the soft-meson limit to produce a monopole structure $[\sim 1/(q^2 - m_N^2)]$, which together with the unknown pole of $N \rightarrow N^*$ will be determined via the best-fitting method.

The OPE is calculated similarly as before by factorizing the η matrix element and the vacuum expectation values. The quark-antiquark component with η appearing in the OPE is written similarly as the pion wave function. Namely, the zeroth moment of the wave function is determined by the soft-meson theorem but the rest of the p_μ dependence is absorbed into the η wave function,

$$\langle 0|\bar{u}(0)i\gamma_5 u(x)|\eta(p)\rangle = -\frac{\langle \bar{u}u\rangle}{f_\eta \sqrt{3}} \int_0^1 dt e^{-itp\cdot x} \varphi_\eta(t) ,$$

$$\langle 0|\bar{d}(0)i\gamma_5 d(x)|\eta(p)\rangle = -\frac{\langle \bar{d}d\rangle}{f_\eta \sqrt{3}} \int_0^1 dt e^{-itp\cdot x} \varphi_\eta(t) . \tag{18}$$

We see a clear distinction from the pion case: the u-quark and d-quark components with η have the *same* overall sign in contrast to Eq. (11). This is because η is an isoscalar particle. The phase convention in fixing the overall sign is consistent with the model by de Swart [3], which has been used in constructing the phenomenological part.

In this definition, the zeroth moment is given by $\int_0^1 dt \varphi_\eta(t) = 1$ as the coefficient is governed by the soft-meson theorem. The use of the soft-meson theorem should be fine in the SU(3) limit. For the second moment, we take $\int_0^1 dt \, t^2 \varphi_\eta(t) = 1/3$ just like the pion case. This is certainly reasonable in the SU(3) limit because in this limit the way of determining the second moment of the pion wave function [18] can be applied equally to the η case. This also makes sense as the OPE satisfies the SU(3) relation as we will see. A question remains as to how one can model this second moment when the SU(3) symmetry is broken. The

second moment multiplied by $p^2 = m_\eta^2$ will contribute to our sum rule. The physical mass of η is much heavier than the pion. Under the same assumption for the second moment, the η matrix element will have an enhancing factor of $m_\eta^2/m_\pi^2 \sim 15.6$ compared to the pion matrix element at the same chiral order. This casts some doubts on the parameterization in Eq. (18). This is a limitation of our current approach and needs to be improved in future. Nevertheless, in this work we will fix the second moment of the η wave function as in the pion case, because the extra modification of the second moment does not affect strongly the SU(3) breaking pattern in the couplings. As we will discuss later, the SU(3) breaking pattern in the couplings is mainly driven by the quark-mass terms in the OPE whose determination requires only the zeroth moment of the wave function.

The OPE for the correlator in Eq. (14) is obtained from Eq. (10) after replacing $\pi^0 \to \eta$. Then the d-quark component of the η wave function has the opposite sign from the pion case, as obtained from the soft-meson theorem. Within the SU(3) limit, except for the overall factor of $1/\sqrt{3}$, this is the only aspect distinct from the pion sum rule. This means that the OPE for ηNN can be obtained from Eq. (12) basically by changing the sign of all the terms except for the last term containing m_0^2. Therefore, the ηNN sum rule is given by

$$
\begin{aligned}
g_{\eta N} m_\eta^2 \lambda_N^2 e^{-m_N^2/M^2}[1 + BM^2] = & \\
\frac{1}{\sqrt{3}} \Bigg\{ & m_\eta^2 M^4 E_0(x) \left[\frac{\langle \bar{q}q \rangle}{12\pi^2 f_\eta} + \frac{3f_{3\eta}}{4\sqrt{2}\pi^2} \right] - \frac{2m_q}{f_\eta}\langle \bar{q}q \rangle^2 M^2 - \frac{m_\eta^2}{72 f_\eta}\langle \bar{q}q \rangle \left\langle \frac{\alpha_s}{\pi}\mathcal{G}^2 \right\rangle \\
& + \frac{2m_q}{3f_\eta}m_0^2\langle \bar{q}q \rangle^2 \Bigg\} .
\end{aligned}
\tag{19}
$$

Here, B denotes the unknown single-pole term, the contribution of $N \to N^*$ plus the one from the derivative of the form factor. In getting the first and third terms, we have used the second moment of the η wave function $\int_0^1 dt\, t^2 \varphi_\eta(t) = 1/3$ but for the second and fourth terms containing the quark-mass we have used the zeroth moment of the η wave function. To extract $g_{\eta N}$, we have to divide both sides by m_η^2. The OPE terms containing m_q will be suppressed by m_η^2 but in the other OPE the m_η^2 dependence will be canceled. As advertised, the SU(3) breaking in $g_{\eta N}$ is mainly driven by the quark-mass terms in which the zeroth moment of the η wave function is used. Therefore, our assumption about the second moment of the η wave function is not a main part in breaking the SU(3) symmetry in the the coupling.

In the SU(3) limit ($f_\eta = f_\pi$, $f_{3\eta} = f_{3\pi}$, $m_\eta^2 = m_\pi^2$), the RHS of Eq. (19) becomes, using the notations introduced in Eq. (13),

$$
\frac{1}{\sqrt{3}}[-\mathcal{O}_1 - \mathcal{O}_2 - \mathcal{O}_3 + \mathcal{O}_4] .
\tag{20}
$$

Compared to the OPE for the πNN case, this reveals interesting aspects of the ηNN coupling. Signs of the first three terms are opposite to those in the pion case. The overall sign of the ηNN coupling should be governed by these terms as the dimension of the operator in last term is the highest in our calculation. This indicates that the sign of the ηNN coupling is opposite to the πNN coupling ! Furthermore, because of this sign difference, $\mathcal{O}_4$ tends to cancel the first two OPE, making the total OPE strength small. This cancellation in addition to the overall suppression factor $1/\sqrt{3}$ leads to the small ηNN coupling in the SU(3) limit.

Another but very important aspect is related to the SU(3) relation for the ηNN coupling. To address this from a simple analysis, postponing a full analysis to the later sections, let us ignore the unknown single-pole terms, A and B, in the sum rules Eqs. (13) and (19). Then the ratio of the two couplings becomes,

$$\frac{g_{\eta N}}{g_{\pi N}} \sim \frac{1}{\sqrt{3}} \frac{-\mathcal{O}_1 - \mathcal{O}_2 - \mathcal{O}_3 + \mathcal{O}_4}{\mathcal{O}_1 + \mathcal{O}_2 + \mathcal{O}_3 + \mathcal{O}_4} \ . \tag{21}$$

On the other hand, the ηNN coupling is known to satisfy the SU(3) relation [3]

$$g_{\eta N} = \frac{g_{\pi N}}{\sqrt{3}}(4\alpha - 1) \ , \tag{22}$$

with $\alpha = F/(F + D)$. By comparing our ratio to this relation, we immediately see that

$$2\alpha \sim \frac{\mathcal{O}_4}{\mathcal{O}_1 + \mathcal{O}_2 + \mathcal{O}_3 + \mathcal{O}_4} \ . \tag{23}$$

Thus, α is closely related to $\mathcal{O}_4$, one of power corrections. Because of the neglected unknown strength, Eq. (23) is not an exact relation for α. Nevertheless, this identification provides an important nature of α. We stress that this identification becomes possible because the sum rules are constructed beyond the chiral limit. If the sum rule is constructed using the soft-meson theorem, the term corresponding to $\mathcal{O}_4$ does not participate in the sum rule and we can not make this kind of identification of α.

Beyond the SU(3) limit, the ηNN sum rule in Eq. (19) has another distinct feature from the πNN sum rule. Even beyond the SU(3) limit, we may still assume that $f_\eta = f_\pi$, $f_{3\eta} = f_{3\pi}$ as they are not expected to be changed substantially. The most important source for the SU(3) breaking is m_η^2, which is much larger than m_π^2. Thus, when both sides of Eq. (19) are divided by m_η^2, the quark mass terms, $\mathcal{O}_2$ and $\mathcal{O}_4$, will be suppressed by the factor m_π^2/m_η^2 from the corresponding terms in Eq. (13). This suppression in addition to the trivial factor of $1/\sqrt{3}$ will be reflected in the physical ηNN coupling.

IV. QCD SUM RULES FOR $\pi\Xi\Xi$ AND $\eta\Xi\Xi$

The QCD sum rules proposed above have interesting features in the SU(3) limit. The OPE is basically the same: the OPE for the ηNN sum rule is different from the πNN case only by the overall factor of $1/\sqrt{3}$ and relative sign of certain terms. This leads to a simple relation for α when the OPE is assumed to be proportional to the coupling. Thus, the two sum rules in the SU(3) limit can be used to determine the F/D ratio. However, for this prediction to be reliable, it is necessary to make a consistency check by calculating other meson-baryon couplings in the SU(3) limit. For this purpose, we construct QCD sum rules for $\pi\Xi\Xi$ and $\eta\Xi\Xi$ in this section.

In constructing the sum rule for $\pi\Xi\Xi$, we use the two-point correlation function of the Ξ interpolating field J_Ξ,

$$\Pi(q, p) = i \int d^4 x e^{iq \cdot x} \langle 0|T[J_\Xi(x)\bar{J}_\Xi(0)]|\pi^0(p)\rangle \ , \tag{24}$$

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with [5]

$$J_\Xi = -\epsilon_{abc}[s_a^T C \gamma_\mu s_b]\gamma_5 \gamma^\mu u_c \ . \tag{25}$$

Since the Ξ interpolating field is obtained from the nucleon interpolating field by replacing u-quark $\to$ s-quark and d-quark $\to$ u-quark, the OPE for Eq. (24) can be obtained directly from Eq. (10) under the similar replacements. Then the term corresponding to the last term in Eq. (10) is zero because it contains

$$\langle 0|\bar{s}(0)i\gamma_5 s(x)|\pi^0\rangle = 0 \ . \tag{26}$$

All other OPE now contain the u-quark component instead of the d-quark component. According to Eq. (11), the u-quark component has the opposite sign of the d-quark.

With these distinctions in mind, we can immediately write the sum rule for the $\pi\Xi\Xi$ coupling,

$$g_{\pi\Xi} \, m_\pi^2 \lambda_\Xi^2 e^{-m_\Xi^2/M^2}[1 + CM^2] =$$
$$m_\pi^2 M^4 E_0(x) \left[\frac{\langle \bar{q}q \rangle}{12\pi^2 f_\pi} + \frac{3f_{3\pi}}{4\sqrt{2}\pi^2} \right] - \frac{2m_s}{f_\pi}\langle \bar{s}s \rangle\langle \bar{q}q \rangle M^2 - \frac{m_\pi^2}{72f_\pi}\langle \bar{q}q \rangle \left\langle \frac{\alpha_s}{\pi}\mathcal{G}^2 \right\rangle \ . \tag{27}$$

Again C denotes unknown single pole term representing the strength $\Xi \to \Xi^*$. Here, we identify a huge SU(3) breaking source in the OPE, m_s. A typical value for m_s is ~ 150 MeV, much larger than m_u or m_d. Another breaking source in the OPE, the strange quark condensate, is only about 20 % smaller than the quark condensate, not badly broken from its SU(3) symmetric limit.

To identify α from this sum rule, we take the SU(3) symmetric limit again. The RHS of Eq. (27) becomes with the notations introduced in Eq. (13),

$$-\mathcal{O}_1 - \mathcal{O}_2 - \mathcal{O}_3 \ . \tag{28}$$

Also, we have $m_\Xi = m_N$. Another phenomenological parameter λ_Ξ^2 also must be equal to the nucleon strength λ_N^2 in the SU(3) limit. This parameter in principle should be determined from the Ξ mass sum rule. The Ξ mass sum rule is different from the nucleon mass sum rule only by the terms containing the s-quark mass [5]. In the SU(3) limit, we have $m_u = m_d = m_s$ and,

$$\lambda_\Xi^2 = \lambda_N^2 \ . \tag{29}$$

Then, as before, by neglecting the unknown single-pole term and taking the ratio with the πNN sum rule, we obtain the relation,

$$\frac{g_{\pi\Xi}}{g_{\pi N}} \sim -\frac{\mathcal{O}_1 + \mathcal{O}_2 + \mathcal{O}_3}{\mathcal{O}_1 + \mathcal{O}_2 + \mathcal{O}_3 + \mathcal{O}_4} \ . \tag{30}$$

Expressing this in terms of α Eq. (23) yields

$$\frac{g_{\pi\Xi}}{g_{\pi N}} \sim 2\alpha - 1 \ . \tag{31}$$

This exactly matches the SU(3) relation proposed in Ref. [3]. Furthermore, from the OPE structure, we see that the sign of $g_{\pi\Xi}$ should be opposite to that of $g_{\pi N}$.

Let us turn our discussion onto the $\eta\Xi\Xi$ sum rule. For this purpose, we use the correlation function,

$$\Pi(q, p) = i \int d^4x e^{iq \cdot x} \langle 0|T[J_\Xi(x)\bar{J}_\Xi(0)]|\eta(p)\rangle \ . \tag{32}$$

In this case, a term corresponding to the last term in Eq. (10) contributes to the sum rule since the s-quark component with η can be written as

$$\langle 0|\bar{s}(0)i\gamma_5 s(x)|\eta\rangle = +\frac{2}{\sqrt{3}}\frac{\langle \bar{s}s\rangle}{f_\eta}\int_0^1 dt e^{-itp \cdot x}\varphi_\eta(t) \ . \tag{33}$$

Again, the coefficient is determined from the soft-meson theorem while the rest of the p-dependence is parametrized in terms of the twist-3 wave function. Only the zeroth moment of this wave function, which is purely governed by the soft-meson theorem, contributes to this sum rule and therefore we don't need to make a further assumption regarding the second moment of this wave function. All other OPE contain the u-quark component with η.

By noting the similarities and distinctions from the $\pi\Xi\Xi$ sum rule, it is straightforward to write down the sum rule for $\eta\Xi\Xi$,

$$g_{\eta\Xi} \ m_\eta^2\lambda_\Xi^2 e^{-m_\Xi^2/M^2}[1 + DM^2] =$$

$$\frac{1}{\sqrt{3}}\Bigg\{m_\eta^2 M^4 E_0(x)\left[\frac{\langle \bar{q}q\rangle}{12\pi^2 f_\eta} + \frac{3f_{3\eta}}{4\sqrt{2}\pi^2}\right] - \frac{2m_s}{f_\eta}\langle \bar{s}s\rangle\langle \bar{q}q\rangle M^2 - \frac{m_\eta^2}{72 f_\eta}\langle \bar{q}q\rangle\left\langle \frac{\alpha_s}{\pi}\mathcal{G}^2\right\rangle$$

$$-\frac{2m_0^2}{3f_\eta}\left[m_s\langle \bar{q}q\rangle\langle \bar{s}s\rangle + m_q\langle \bar{s}s\rangle^2\right] \ \Bigg\} \ . \tag{34}$$

Again, we identify the SU(3) breaking sources, m_s, $\langle \bar{s}s\rangle$, f_η and $f_{3\eta}$. The most important source for the SU(3) breaking is m_s. The terms containing m_s are obtained by using the zeroth moment of the η wave function whose value is well-fixed by the soft-meson theorem.

Using the notations introduced in Eq. (13), the RHS of Eq. (34) in *the SU(3) limit* becomes

$$\frac{1}{\sqrt{3}}[-\mathcal{O}_1 - \mathcal{O}_2 - \mathcal{O}_3 - 2\mathcal{O}_4] \ . \tag{35}$$

This OPE indicates that $g_{\eta\Xi}$ has the opposite sign of $g_{\pi N}$. By neglecting the unknown constant D within the SU(3) limit and taking the ratio with the πNN sum rule, we obtain

$$\frac{g_{\eta\Xi}}{g_{\pi N}} \sim -\frac{1}{\sqrt{3}}\frac{\mathcal{O}_1 + \mathcal{O}_2 + \mathcal{O}_3 + 2\mathcal{O}_4}{\mathcal{O}_1 + \mathcal{O}_2 + \mathcal{O}_3 + \mathcal{O}_4} \ . \tag{36}$$

In terms of α Eq. (23), the ratio becomes

$$\frac{g_{\eta\Xi}}{g_{\pi N}} \sim -\frac{1}{\sqrt{3}}(1 + 2\alpha) \tag{37}$$

matching the SU(3) relation of Ref. [3] exactly again. Therefore, our identification of α as given in Eq. (23) suggests that our OPE for the $\pi\Xi\Xi$ and $\eta\Xi\Xi$ couplings are consistent with the SU(3) relations.

V. QCD SUM RULES FOR $\pi\Sigma\Sigma$ AND $\eta\Sigma\Sigma$

Another examples where our formalism is directly applicable are the $\pi\Sigma\Sigma$ and $\eta\Sigma\Sigma$ couplings. To calculate the couplings, we simply substitute the nucleon interpolating field with the Σ interpolating field, $J_p \to J_\Sigma$. For the $\pi\Sigma\Sigma$ sum rule, we need to consider

$$i \int d^4x e^{iq\cdot x} \langle 0|T[J_\Sigma(x)\bar{J}_\Sigma(0)]|\pi^0(p)\rangle \ , \tag{38}$$

with [5]

$$J_\Sigma = \epsilon_{abc}[u_a^T C\gamma_\mu u_b]\gamma_5\gamma^\mu s_c \ . \tag{39}$$

This interpolating field can be obtained from the nucleon interpolating field by replacing,

$$\text{d} - \text{quark} \to \text{s} - \text{quark} \ . \tag{40}$$

This means that the OPE in this case can be obtained from Eq. (10) by the same replacement and, as the s-quark component with a pion is zero, only the term corresponding to the last term in Eq. (10) will give a nonzero contribution. Thus, the sum rule for $\pi\Sigma\Sigma$ is

$$g_{\pi\Sigma} \, m_\pi^2 \lambda_\Sigma^2 e^{-m_\Sigma^2/M^2}[1 + EM^2] = \frac{m_0^2}{3f_\pi}\left[m_q\langle\bar{s}s\rangle\langle\bar{q}q\rangle + m_s\langle\bar{q}q\rangle^2\right] \ . \tag{41}$$

As m_s changes substantially in the SU(3) breaking limit, the OPE undergoes a substantial change as we go from the SU(3) symmetric limit to its breaking limit. In the SU(3) symmetric limit $[m_s = m_q, \langle\bar{s}s\rangle = \langle\bar{q}q\rangle]$, the RHS, which is then equal to $\mathcal{O}_4$, satisfies the SU(3) relation if Eq. (23) is used, that is,

$$\frac{g_{\pi\Sigma}}{g_{\pi N}} \sim 2\alpha \ . \tag{42}$$

Note, the OPE is positive, $\alpha > 0$.

Now, for the $\eta\Sigma\Sigma$ case, we can similarly proceed using the correlator

$$i \int d^4x e^{iq\cdot x} \langle 0|T[J_\Sigma(x)\bar{J}_\Sigma(0)]|\eta(p)\rangle \ . \tag{43}$$

In this case, similarly as the $\eta\Xi\Xi$ case, the s-quark component have a nonzero value with η [see Eq.(33).]. Straightforward calculations yield,

$$g_{\eta\Sigma} \, m_\eta^2 \lambda_\Sigma^2 e^{-m_\Sigma^2/M^2}[1 + FM^2] =$$
$$\frac{1}{\sqrt{3}}\left\{ -2m_\eta^2 M^4 E_0(x)\left[\frac{\langle\bar{s}s\rangle}{12\pi^2 f_\eta} + \frac{3f_{3\eta}}{4\sqrt{2}\pi^2}\right] + \frac{4m_q}{f_\eta}\langle\bar{s}s\rangle\langle\bar{q}q\rangle M^2 + \frac{m_\eta^2}{36f_\eta}\langle\bar{s}s\rangle\left\langle\frac{\alpha_s}{\pi}\mathcal{G}^2\right\rangle \right.$$
$$\left. +\frac{m_0^2}{3f_\eta}\left[m_q\langle\bar{s}s\rangle\langle\bar{q}q\rangle + m_s\langle\bar{q}q\rangle^2\right]\right\} \ . \tag{44}$$

In getting the first and third terms in the OPE, we have used the second moment of the η wave function, $1/3$. This assumption is made in analogy with the pion wave function. However, since we are dealing with the s-quark component with η in this case, our assumption

for the η wave function for the second moment has been further extended. Once again in the SU(3) limit, the RHS takes the form

$$\frac{1}{\sqrt{3}} \left[2\mathcal{O}_1 + 2\mathcal{O}_2 + 2\mathcal{O}_3 + \mathcal{O}_4 \right] \, , \tag{45}$$

which, when combined with Eq. (23), yields the SU(3) relation for $\eta\Sigma\Sigma$,

$$\frac{g_{\eta\Sigma}}{g_{\pi N}} \sim \frac{2}{\sqrt{3}}(1 - \alpha) \, , \tag{46}$$

matching again the SU(3) relation of de Swart [3].

VI. ANALYSIS IN THE SU(3) LIMIT – DETERMINATION OF THE F/D RATIO

So far, we have presented the sum rules for πNN, ηNN $\pi\Xi\Xi$, $\eta\Xi\Xi$, $\pi\Sigma\Sigma$ and $\eta\Sigma\Sigma$ beyond the chiral limit. As far as the OPE is concerned, the sum rules satisfy the SU(3) relations, indicating that they are not independent. They are related through the SU(3) rotations. One assumption made to the second moment of the η wave function, which is taken to be the same as the one from the pion wave function $\int_0^1 dt \, t^2\varphi_\eta(t) = 1/3$, is valid in this consideration, perhaps motivating its use even beyond the SU(3) symmetric limit. From the consistency with the SU(3) relations, we have identified the OPE responsible for the F/D ratio. In reaching this identification, it is important to construct the QCD sum rules beyond the chiral limit.

In this section, we determine the F/D ratio from the sum rules Eqs. (13) (19) (27) (34) (41) (44). As the F/D ratio is a parameter defined in the SU(3) limit, we consider the sum rules in *the SU(3) limit*. In this limit, we have exact relations,

$$\lambda_N^2 = \lambda_\Xi^2 = \lambda_\Sigma^2 \, , \tag{47}$$

as all the baryonic mass sum rules are equal [5]. In our analysis, we use the standard QCD parameters,

$$\langle \bar{q}q \rangle = -(0.23 \text{ GeV})^3 \, ; \quad \left\langle \frac{\alpha_s}{\pi}\mathcal{G}^2 \right\rangle = (0.33 \text{ GeV})^4 \, ; \quad m_0^2 = 0.8 \text{ GeV}^2 \, . \tag{48}$$

We arrange each sum rule into the form

$$a + bM^2 = f(M^2) \tag{49}$$

by dividing both sides of each sum rule by the meson mass squared and the exponential factor in the phenomenological side. bM^2 indicates the contributions from $N \rightarrow N^*$ (or $\Xi \rightarrow \Xi^*$, $\Sigma \rightarrow \Sigma^*$). Specifically, in the πNN sum rule Eq. (13), we divide both sides by $m_\pi^2 \, e^{-m_N^2/M^2}$. Thus, in this sum rule, $a = g_{\pi N}\lambda_N^2$, $b = g_{\pi N}\lambda_N^2 A$ and the RHS becomes

$$f(M^2) = \frac{\mathcal{O}_1 + \mathcal{O}_2 + \mathcal{O}_3 + \mathcal{O}_4}{m_\pi^2 \, e^{-m_N^2/M^2}} \, . \tag{50}$$

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Recall that $\mathcal{O}_1$ and $\mathcal{O}_3$ contain m_π^2. The quark-mass m_q in $\mathcal{O}_2$ and $\mathcal{O}_4$, with the use of the Gell-Mann−Oakes−Renner relation, can be converted to m_π^2, which is then canceled with another m_π^2 in the denominator. Therefore, the sum rule in its final form does not depend on the quark-mass or m_π^2. The parameters a and b will be determined by fitting $f(M^2)$ with a straight line within a Borel window. Similarly constructed Borel curves for all sum rules $f(M^2)$ are shown in Fig. 1, Fig. 2 and Fig. 3. A common feature is that the continuum threshold does not affect the Borel curve much.

In the fitting process, we need to choose an appropriate Borel window. As can be seen from the figures, depending on the Borel window we choose, we would get different values for the parameters a and b. But as long as the Borel window is chosen for $M_{min}^2 \geq 0.7$ GeV^2, all Borel curves are relatively well-fitted by straight lines, reducing the sensitivity to the Borel window. Nonetheless, a more important claim about the Borel windows can be made from the OPE structure of each sum rule. As we have demonstrated in earlier sections, the OPE of each sum rule in the SU(3) limit satisfies the SU(3) relation. This means that as far as the OPE is concerned, all sum rules are related by the SU(3) rotations: they are not independent. A point in a Borel curve at a specific Borel mass is transformed under the SU(3) rotation to a point of an other Borel curve defined at the same Borel mass. It implies that once a Borel window is fixed in one sum rule, the same Borel window must be applied to the other sum rules as they are not independent. This claim can be justified if the extracted F/D ratio is independent of the sum rule considered.

We fix the common Borel window from the πNN sum rule. According to the analysis in Ref. [10], the Borel window $0.65 \leq M^2 \leq 1.24$ GeV^2 is obtained as a common window for the πNN sum rule and the chiral-odd nucleon mass sum rule. It provides $g_{\pi N}$ close to its empirical value. Using this Borel window to other sum rules, we determine the parameters a and b from each Borel curve. The parameter a is obtained from the intersection of the best-fitting curve with the y-axis. The slope of the best fitting curve yields the parameter b. They are listed in table I. From the first column, we see that the signs of the couplings obtained from the best-fitting method are consistent with our naive analysis given in Eqs. (21), (31), (36), (42), (46). Specifically,

$$g_{\eta N}^{(S)} < 0 ; \quad g_{\pi \Xi}^{(S)} < 0 ; \quad g_{\eta \Xi}^{(S)} < 0$$
$$g_{\pi \Sigma}^{(S)} > 0 ; \quad g_{\eta \Sigma}^{(S)} > 0 . \tag{51}$$

Here the superscript (S) indicates the couplings in the SU(3) limit. These signs restrict the range of $\alpha[\equiv F/(F+D)]$,

$$0 < \alpha < \frac{1}{4} \rightarrow 0 < \frac{F}{D} < \frac{1}{3} . \tag{52}$$

This is a constraint for the F/D ratio to be satisfied within our QCD sum rule analysis.

From the ratios provided in the fourth column, we determine α from each sum rule since we know the SU(3) relation for each coupling [3][3]. The five ratios presented in table I

---

[3] Note, according to Eq. (47), all strengths of the interpolating fields to the baryons are the same. Therefore, we can think of the parameter a as the coupling times the multiplicative factor common to all sum rules.

consistently give $\alpha = 0.175$. This justifies the use of the common Borel window for all sum rules. The F/D ratio from this value is 0.212. To see the sensitivity to the Borel window, we blindly shift the common Borel window to $0.9 \leq M^2 \leq 1.5 \, \mathrm{GeV}^2$. Of course, in this window, we would not obtain the πNN coupling consistent with its empirical value. Nevertheless, the F/D ratio from this Borel window is 0.196, which is not far from the one above. From this analysis, we can safely claim that

$$F/D \sim 0.2 \ . \tag{53}$$

This is about a factor of 3 smaller than what the SU(6) predicts, $F/D = 2/3$ or the more recent value [20], $F/D \sim 0.57$. It is interesting to see that these values from other studies even badly violate the constraint from our study Eq. (52). The small F/D ratio makes sense in our approach because it is generated by the highest dimensional operator in the OPE.

From the ratios in table I, we can also calculate the couplings in the SU(3) limit if the πNN coupling is given. Using $g_{\pi N} = 13.4$, we obtain from the fourth column in table I,

$$g_{\eta N}^{(S)} = -2.3 \ ; \quad g_{\pi \Xi}^{(S)} = -8.7 \ ; \quad g_{\eta \Xi}^{(S)} = -10.5$$
$$g_{\pi \Sigma}^{(S)} = 4.7 \ ; \quad g_{\eta \Sigma}^{(S)} = 12.8 \ . \tag{54}$$

Note, the signs of the couplings are relative to $g_{\pi N}$. Each coupling, when it is combined with the first column of table I, consistently yields the strength,

$$\lambda_N^2 \sim 3.5 \times 10^{-4} \, \mathrm{GeV}^6 \ , \tag{55}$$

which of course should be equal to λ_Ξ^2 and λ_Σ^2 in the SU(3) limit. This is the first determination of λ_N without explicit use of the nucleon mass sum rules. For later discussions, we close this section by listing the ratios of η-baryon couplings over pion-baryon couplings obtained in the SU(3) limit,

$$\frac{g_{\eta N}^{(S)}}{g_{\pi N}} = -0.17 \ ; \quad \frac{g_{\eta \Xi}^{(S)}}{g_{\pi \Xi}^{(S)}} = 1.2 \ ; \quad \frac{g_{\eta \Sigma}^{(S)}}{g_{\pi \Sigma}^{(S)}} = 2.71 \ . \tag{56}$$

Note, we have not put the superscript on $g_{\pi N}$ because it is independent of whether or not the SU(3) limit is taken. The magnitude of $g_{\eta N}^{(S)}$ is a lot smaller than $g_{\pi N}^{(S)}$, while $g_{\eta \Xi}^{(S)}$ and $g_{\eta \Sigma}^{(S)}$ are larger than the corresponding pion-baryon couplings.

VII. MESON-BARYON COUPLINGS–QUALITATIVE ANALYSIS

Having established the F/D ratio in the SU(3) limit, we now move on to an analysis *beyond* the SU(3) limit. Due to the symmetry breaking, we have different baryonic masses, meson masses, and $\lambda_N \neq \lambda_\Xi \neq \lambda_\Sigma$. Moreover, some QCD parameters change from their values in the symmetric limit. This means that all sum rules are not simply related by the SU(3) rotations and a separate Borel analysis is necessary in each sum rule. However, in predicting the couplings, we have several limitations. First, as mentioned, the assumption used for the second moment of the η wave function may not be valid in the breaking limit. We speculate from the soft-meson limit that corrections to this assumption are small but

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the soft-meson theorem may not be strictly valid as mesons become heavier. In addition, we do not have clear restrictions on the QCD parameters f_η, $f_{3\eta}$, m_s and $\langle \bar{s}s \rangle$. The η decay constant f_η is known to be about 20 % larger from its SU(3) value but $f_{3\eta}$ is not well under control. The standard values for m_s and $\langle \bar{s}s \rangle$ are

$$m_s = 150 \text{ MeV} ; \quad \langle \bar{s}s \rangle = 0.8 \langle \bar{q}q \rangle . \tag{57}$$

Even though we will use these values in this work, there are still some on-going discussions on these values [21,22]. Moreover, there is an additional source of the SU(3) breaking driven by the $\eta - \eta'$ mixing. Some studies [23] suggest that η is almost η_8 but there is no consensus on this point. Therefore, our analysis in this section should be regards as "qualitative".

We will take $f_\pi = f_\eta$, $f_{3\pi} = f_{3\eta}$, and ignore the mixing between $\eta - \eta'$. The most important source for the SU(3) breaking in the QCD side is the strange quark mass m_s. Compared to the up-(or down-)quark mass, m_s is very large, more than 20 times of m_q. In the hadronic side, baryonic masses, meson masses, and strength of the interpolating fields to the physical baryons will be changed in the breaking. In this section, we will study how these breakings are reflected in the couplings.

Figure 4 shows the Borel curve for the ηNN coupling. It is almost a straight line with respect to M^2. Recall that this curve is obtained by dividing Eq. (19) with $m_\eta^2 = (547 \text{ MeV})^2$ in addition to the exponential factor of $e^{-m_N^2/M^2}$. The quark-mass terms in the OPE will be suppressed by the factor m_π^2/m_η^2 compared to the corresponding terms in the SU(3) symmetric case. This will shift the Borel curve upward as shown in Fig. 4, which gives small value $a \sim -0.00022$ in table II. Since a is very small, the relative sensitivity to the continuum threshold or the Borel window becomes large in this case. Nevertheless, by dividing it with the a from the πNN sum rule and using the empirical value $g_{\pi N} = 13.4$, we obtain $g_{\eta N} = -0.63$, negative value but its magnitude practically consistent with zero.

A remarkable breaking effect can be observed in the $\pi \Xi \Xi$ Eq. (27). As m_s involved in the OPE is substantially increased in the SU(3) breaking limit, the quark-mass term will be enhanced by a factor of 20 while other OPE terms remain the same. A further enhancement of the Borel curve comes from the exponential factor $e^{m_\Xi^2/M^2}$. The resulting Borel curve is shown in Fig. 5. Around the resonance mass, $m_\Xi^2 \sim 1.73 \text{ GeV}^2$, the Borel curve is almost flat, indicating that the unknown single pole term is small. This kind of strong enhancement of the Borel curve is not observed in the $\eta \Xi \Xi$ case. In this case, the dividing factor m_η^2, much larger than m_π^2, compensates the strong enhancement coming from m_s. Similar behaviors can be observed in Fig. 6 for $\pi \Sigma \Sigma$ and $\eta \Sigma \Sigma$. Here the enhancement of the $\pi \Sigma \Sigma$ Borel curve due to m_s is not so strong as $\pi \Xi \Xi$ because the OPE contains only one term.

The best fitting parameters for a and b from each sum rule are listed in table II. Recall that the numbers in the first column is the couplings multiplied by the strengths of the interpolating fields to the baryons, λ_N^2 (or λ_Ξ^2 or λ_Σ^2 depending on the sum rule). Since the SU(3) symmetry is broken, the strengths are not equal. To see the SU(3) breaking effects, we compare the first column in table I with the one in table II and obtain the ratios

$$\frac{g_{\eta N}^{(B)}}{g_{\eta N}^{(S)}} = 0.275 ; \quad \frac{g_{\pi \Xi}^{(B)}[\lambda_\Xi^{(B)}]^2}{g_{\pi \Xi}^{(S)} \lambda_N^2} = 33.6 ; \quad \frac{g_{\eta \Xi}^{(B)}[\lambda_\Xi^{(B)}]^2}{g_{\eta \Xi}^{(S)} \lambda_N^2} = 1.63 ,$$

$$\frac{g_{\pi \Sigma}^{(B)}[\lambda_\Sigma^{(B)}]^2}{g_{\pi \Sigma}^{(S)} \lambda_N^2} = 12.34 ; \quad \frac{g_{\eta \Sigma}^{(B)}[\lambda_\Sigma^{(B)}]^2}{g_{\eta \Sigma}^{(S)} \lambda_N^2} = 0.447 . \tag{58}$$

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The superscript (B) denotes that the couplings in the SU(3) breaking limit. In the denominators, we have λ_N^2 instead of the strength for the corresponding interpolating field because all strengths are the same in the SU(3) limit. In the ratio for the ηNN coupling, the strength of the nucleon interpolating field λ_N^2 has been canceled as it is blind to the SU(3) symmetry. Most ratios show the huge SU(3) breaking. This is mainly driven by m_s in the QCD side and the meson masses in the phenomenological side. To see the breaking reflected only in the couplings, we need to eliminate the strength of each baryon to its interpolating field. A recent study [24] suggests that the strength λ_B^2 for the baryon B with mass M_B scales like $\lambda_B^2 \sim C M_B^6$ with some constant C. Of course this scaling needs to be confirmed by further studies but nevertheless using this information in our sum rules, we obtain the following ratios,

$$\frac{g_{\eta N}^{(B)}}{g_{\eta N}^{(S)}} = 0.275 \; ; \quad \frac{g_{\pi\Xi}^{(B)}}{g_{\pi\Xi}^{(S)}} = 4.45 \; ; \quad \frac{g_{\eta\Xi}^{(B)}}{g_{\eta\Xi}^{(S)}} = 0.22 \; ,$$

$$\frac{g_{\pi\Sigma}^{(B)}}{g_{\pi\Sigma}^{(S)}} = 2.99 \; ; \quad \frac{g_{\eta\Sigma}^{(B)}}{g_{\eta\Sigma}^{(S)}} = 0.11 \; . \tag{59}$$

These clearly show huge SU(3) breaking in the couplings. Ref. [25] provides different values for the strengths λ_Σ and λ_Ξ, which do not seem to satisfy the scaling [24]. These strengths from Ref. [25] if used in our work provide larger values for the ratios in Eq. (59). In this case, the SU(3) breaking in the couplings becomes more drastic. Therefore, the scaling $\lambda_B^2 \sim C M_B^6$ provides a mild SU(3) breaking in the couplings even though it is still large. By combining Eq. (59) with the couplings in the SU(3) limit Eq. (54), we calculate the couplings beyond the SU(3) limit and present them in table III as well as the ones in the SU(3) limit. The suppression of $g_{\eta NN}$ seems to support the results of Ref. [26] and is severe than the one from Bonn potential [1]. The couplings with hyperons do not agree with Nijmegen potential model [2]. Nijmegen potential is based on SU(3) symmetry and the SU(3) breaking enters in the model perturbatively. The couplings determined by fitting the experimental YN scatterings seem to be consistent with SU(3) symmetry [2]. However, the hyperon-interaction data are rather scarce to determine the couplings reliably and it will be interesting in future to see how our findings affect the analysis of Nijmegen potential. To go that direction however we need to make a further study of our approach and solidify our results. As we have mentioned, our results beyond the SU(3) limit at this stage should be regarded as qualitative due to certain assumptions made in the analysis.

To see the SU(3) breaking without explicit use of the informations from Ref. [24], we also calculate the ratios,

$$\frac{g_{\eta N}^{(B)}}{g_{\pi N}} = -0.05 \; ; \quad \frac{g_{\eta\Xi}^{(B)}}{g_{\pi\Xi}^{(B)}} = 0.06 \; ; \quad \frac{g_{\eta\Sigma}^{(B)}}{g_{\pi\Sigma}^{(B)}} = 0.1 \; . \tag{60}$$

When these ratios are compared with the corresponding ones in the SU(3) limit Eq. (56), we again notice that the SU(3) breaking effects are huge in the couplings. The three ratios are consistently smaller in magnitude than their corresponding values in the SU(3) symmetric limit. The suppression is severe in the ratios, $g_{\eta\Xi}/g_{\pi\Xi}$ and $g_{\eta\Sigma}/g_{\pi\Sigma}$. The pion-baryon couplings are enhanced by the strange-quark mass in the OPE, while in the η-baryon couplings,

this enhancement is reduced by the overall dividing factor m_η^2. When the ambiguity in the form factor is considered, the η-baryon couplings can be increased by 13 %. More softer cut-off like $\Lambda \sim 1$ GeV increases the coupling slightly more. But even so, it does not change our conclusion that the SU(3) breaking is huge. Further changes are expected from the uncertainties in m_s, $\eta - \eta'$ mixing and so forth. Nevertheless, the trend that we have observed, especially the claim that there is huge SU(3) breaking in the the ratios Eqs. (56) (60), is expected to be maintained even if we take into account the limitations of our approaches.

Another important finding in our work is the relative signs of the couplings with respect to $g_{\pi N} > 0$. The signs provided in Eq. (51) are preserved even in the SU(3) breaking limit.

$$
\begin{aligned}
g_{\eta N}^{(B)} < 0 \; ; \quad g_{\pi \Xi}^{(B)} < 0 \; ; \quad g_{\eta \Xi}^{(B)} < 0 \\
g_{\pi \Sigma}^{(B)} > 0 \; ; \quad g_{\eta \Sigma}^{(B)} > 0 \; .
\end{aligned}
\tag{61}
$$

For the ηNN coupling, the sign should be opposite of $g_{\pi N}$, because the highest dimensional operator should be smaller in magnitude than the leading OPE terms. For the other couplings, the signs can be simply read off from each OPE. Certainly, in the SU(3) limit, these signs are crucial in explaining the consistency of each OPE with the SU(3) relation for the corresponding coupling.

VIII. SUMMARY

In this work, we have developed QCD sum rules beyond the chiral limit for the diagonal meson-baryon couplings, πNN, ηNN, $\pi \Xi\Xi$, $\eta \Xi\Xi$, $\pi \Sigma\Sigma$ and $\eta \Sigma\Sigma$. We have assumed the second moment of the twist-3 η wave function to be the same as the one from the pion wave function. This should be exact in the SU(3) limit. The most important finding in this work is that the OPE structures match the SU(3) relations for the couplings in the SU(3) limit. Going beyond the chiral limit is crucial for this identification. Thus, we have identified the OPE responsible for the F/D ratio. From a Borel analysis, it was found to be around $F/D \sim 0.2$ strongly disagreeing with the SU(6) prediction. In future, it will be useful to do similar calculations for other Dirac structures [6,9] and see if consistent results can be obtained. In the SU(3) breaking case, we have calculated the ratios, the couplings divided by the corresponding values in the SU(3) limit. The ratios, even though the scaling law is used for the baryon strength $\lambda_B^2 \sim M_B^6$, suggest that the couplings violate the SU(3) symmetry strongly. Also we have presented the ratios of η-baryon couplings to pion-baryon couplings, which does not require an assumption for the strength λ_B^2. We found that η-baryon couplings are much smaller than pion-baryon couplings. Compared to the corresponding ratio in the SU(3) limit, we have found that huge SU(3) breaking exists in the couplings. In future, it will be interesting to explore this aspect in hyperon-nucleon interactions.

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TABLES

TABLE I. The best-fitting values for the parameters a and b in the SU(3) symmetric limit are listed for each sum rule within the Borel window $0.65 \leq M^2 \leq 1.24$ GeV^2. In the fourth column, we present ratios of each coupling divided by $g_{\pi N}$, which are directly related to the F/D ratio. The obtained value for the F/D ratio is 0.212.

	a (GeV^6)	b (GeV^4)	coupling/$g_{\pi N}$
πNN	0.00464	0.00084	1
ηNN	-0.0008	-0.00146	-0.172
$\pi \Xi\Xi$	-0.00302	-0.00171	-0.651
$\eta \Xi\Xi$	-0.00362	-0.00002	-0.78
$\pi \Sigma\Sigma$	0.00163	-0.00087	0.351
$\eta \Sigma\Sigma$	0.00442	0.00147	0.953

TABLE II. The best-fitting values for the parameters a and b beyond the SU(3) symmetric limit are listed for each sum rule within the Borel windows taken around the resonance masses. For $\pi \Sigma\Sigma$ case, there is no continuum contribution because the OPE does not contain the perturbative part.

	a (GeV^6)	b (GeV^4)	Borel window (GeV^2)	S_0 (GeV^2)
ηNN	-0.00022	-0.00097	$0.65 - 1.24$	2.07
$\pi \Xi\Xi$	-0.1015	-0.00202	$1.53 - 1.93$	3.
$\eta \Xi\Xi$	-0.0059	-0.001	$1.53 - 1.93$	3.
$\pi \Sigma\Sigma$	0.02012	-0.0072	$1.21 - 1.61$	-
$\eta \Sigma\Sigma$	0.00202	0.00178	$1.21 - 1.61$	3.

TABLE III. Meson-baryon diagonal couplings in the SU(3) limit and beyond the SU(3) limit are presented. As we have discussed in the text, the values beyond the SU(3) limit should be regarded as qualitative. The πNN coupling in the first line is the empirical value.

	SU(3) limit	Beyond the SU(3) limit
$g_{\pi N}$	13.4	13.4
$g_{\eta N}$	-2.3	-0.63
$g_{\pi \Xi}$	-8.7	-38.7
$g_{\eta \Xi}$	-10.5	-2.3
$g_{\pi \Sigma}$	4.7	14.1
$g_{\eta \Sigma}$	12.8	1.4

FIG. 1. The Borel mass dependence of $a + bM^2$ for πNN and ηNN sum rules in the SU(3) symmetric case. The continuum threshold $S_0 = 2.07$ GeV^2, corresponding to the Roper resonance, is used for the solid lines. To see the sensitivity to the continuum threshold, the dashed-lines with $S_0 = 2.57$ GeV^2 are also plotted. In the case of πNN, the continuum gives 2% corrections at $M^2 = 1$ GeV^2.
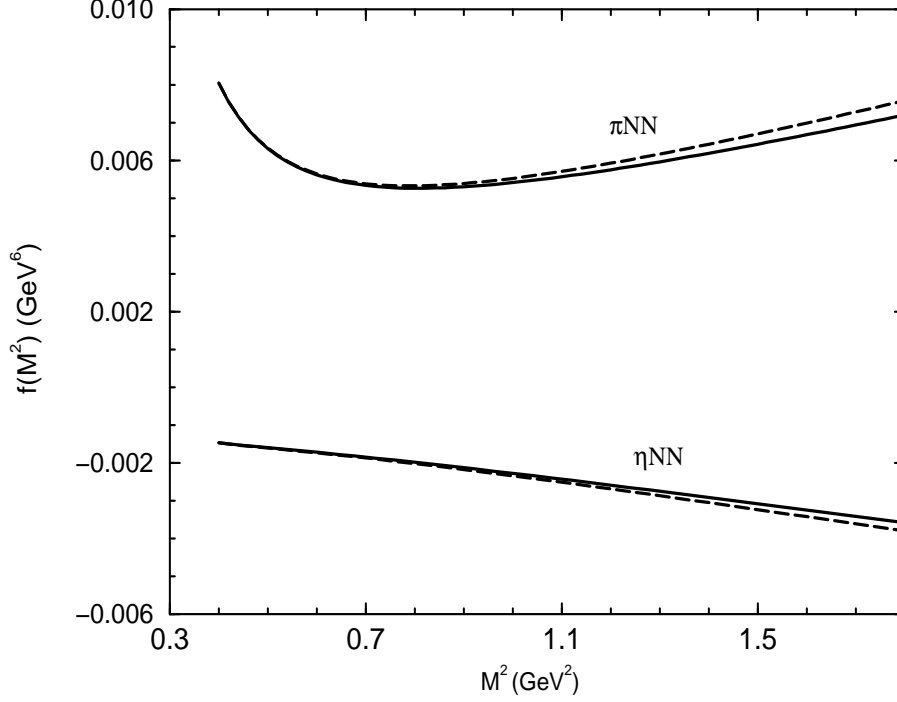

FIG. 2. The Borel curves for $\pi \Xi\Xi$ and $\eta \Xi\Xi$ in the SU(3) limit. The dashed lines show the sensitivity to the continuum threshold.
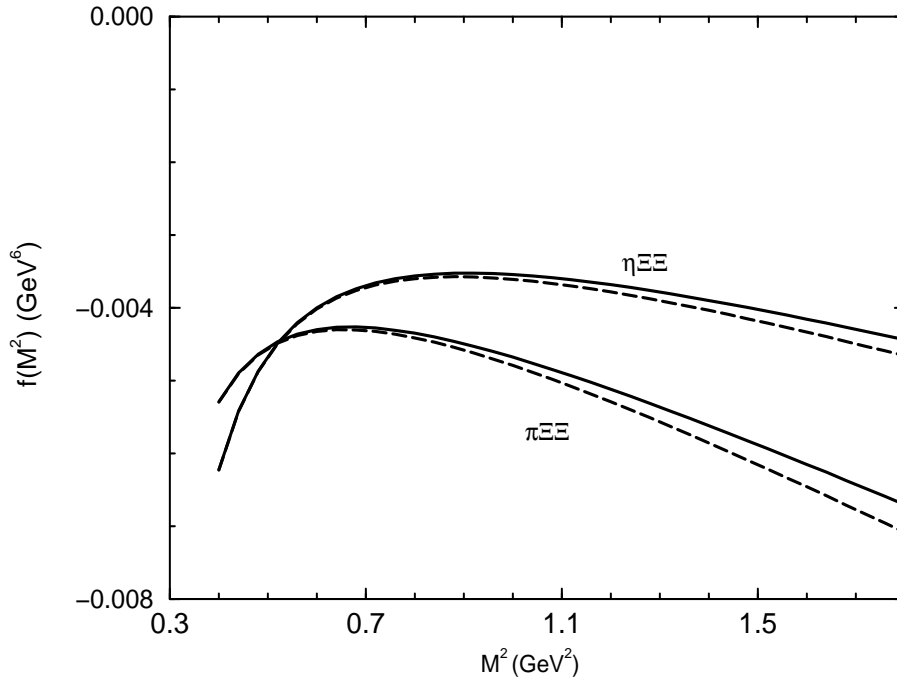
FIG. 3. The Borel curves for $\pi\Sigma\Sigma$ and $\eta\Sigma\Sigma$ in the SU(3) limit. In the curve for $\pi\Sigma\Sigma$, there is no sensitivity to the continuum threshold as the OPE has only the power corrections.
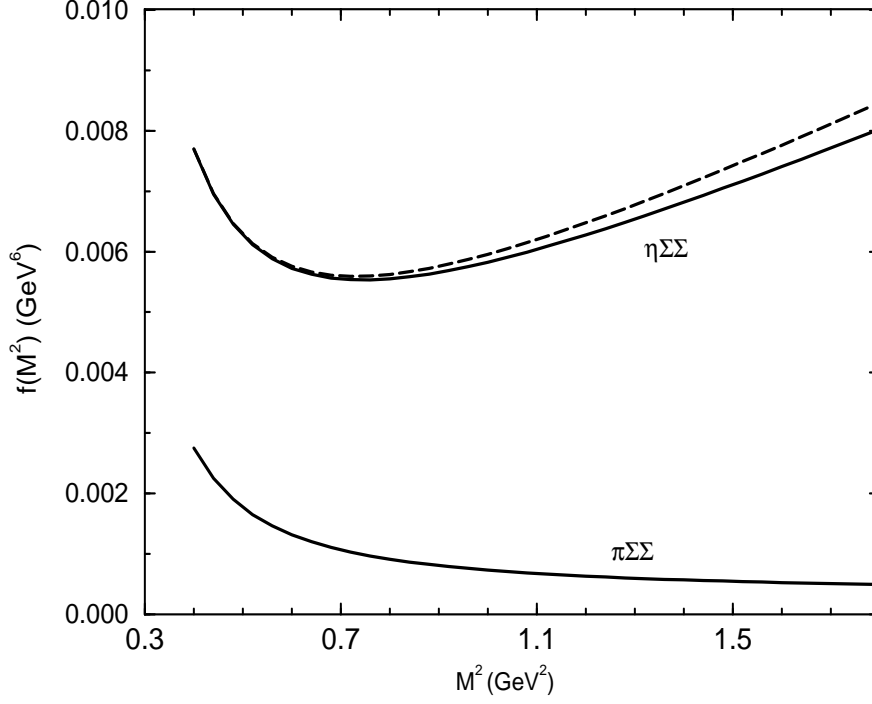

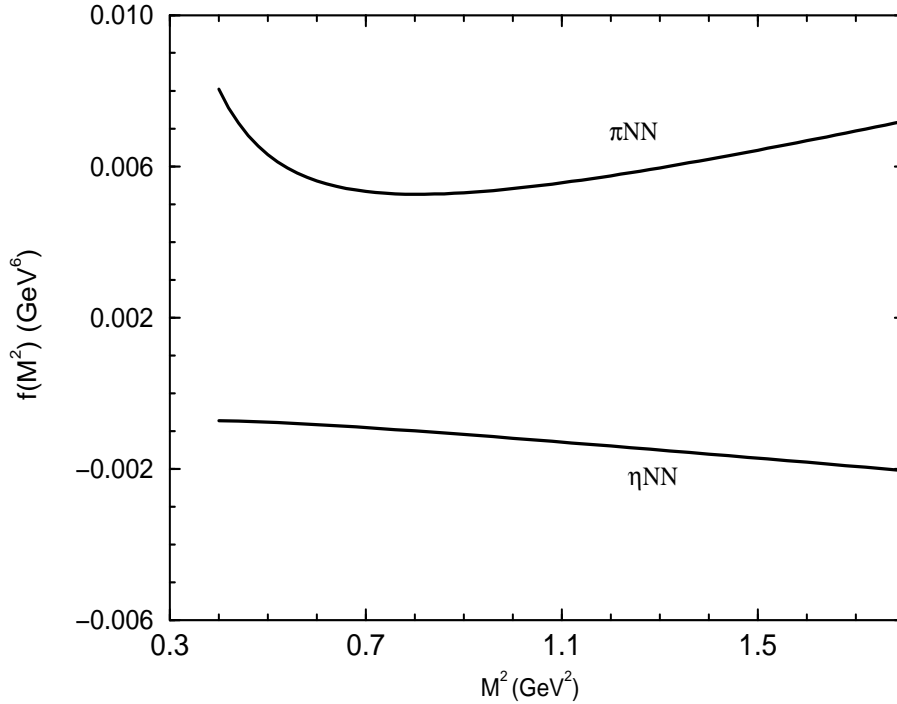FIG. 4. The Borel curve for ηNN beyond the SU(3) limit. The πNN Borel curve is also shown for comparison.

FIG. 5. The Borel curves for $\pi\Xi\Xi$ and $\eta\Xi\Xi$ beyond the SU(3) limit. The continuum threshold $S_0 = 3$ GeV^2 is used. Note, compared with the SU(3) symmetric case in Fig.(3), the scale in y-axis is much larger here.
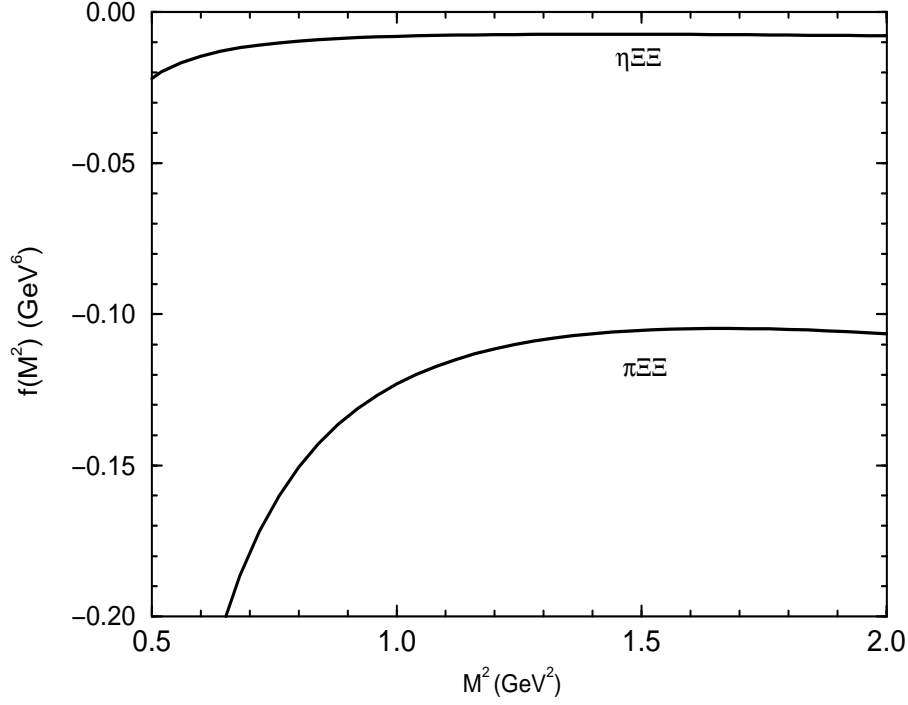

FIG. 6. The Borel curves for $\pi\Sigma\Sigma$ and $\eta\Sigma\Sigma$ beyond the SU(3) limit. The continuum threshold $S_0 = 3$ GeV^2 is used.